Agribusiness Analysis and Forecasting

Mixed Marginal Distributions

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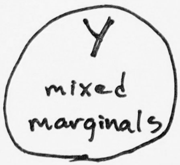
Reveiw

- Critical to appropriately reflect dependence among multiple variables
- MV normal is one approach to reflecting dependence
- Under MV Normal, all marginal distributions in the system are normal (not ideal)
- Under MV Normal, dependence among variables is strictly linear (not ideal)

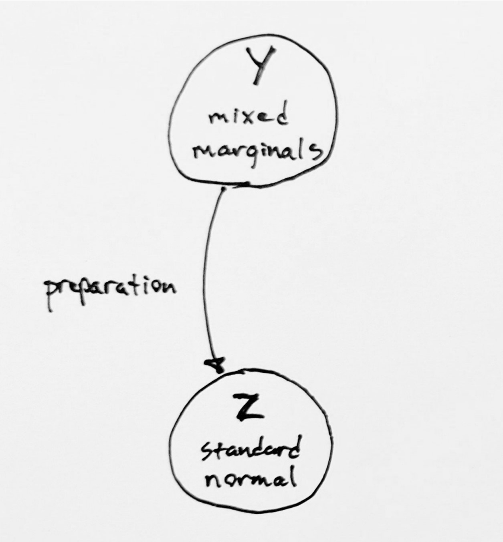
Mixed Marginals

- This topic: alternative approach to modeling dependence
- The key is separating the modeling of the individual marignal distributions and the modeling of the dependence
- The dependence is modeled using MV normal

Overview



Overview



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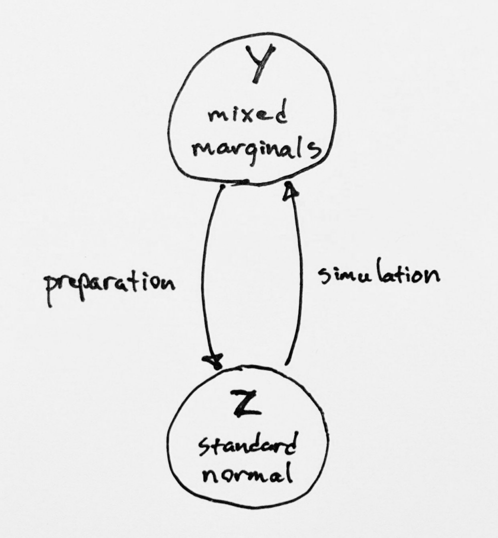
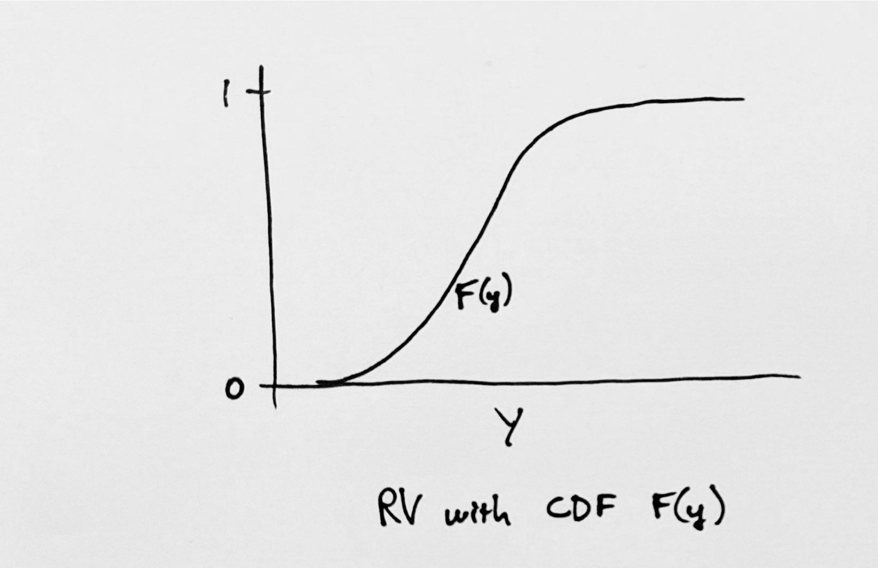

Y
mixed
marginals
preparation
simulation
Z
standard
normal

Review of CDF and Inverse CDF

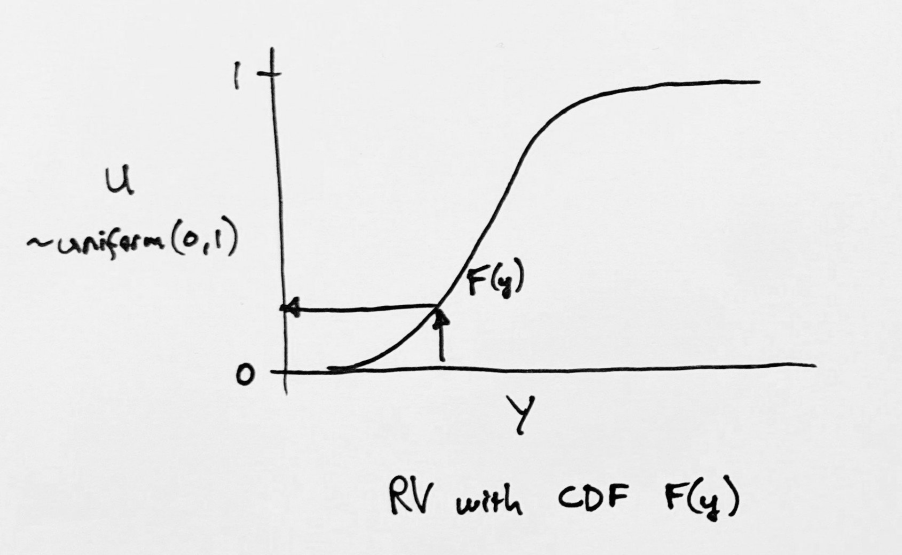
Y

RV with CDF $F(y)$

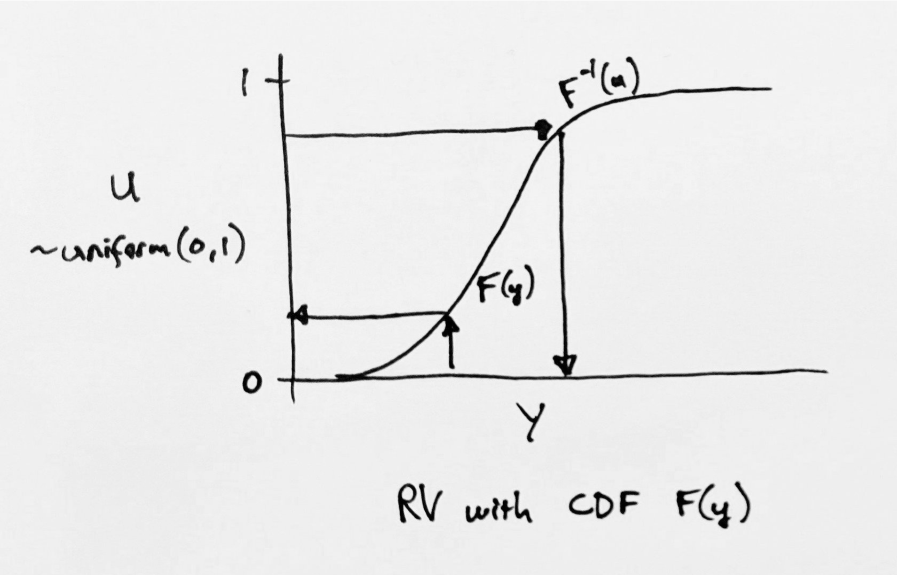
Review of CDF and Inverse CDF



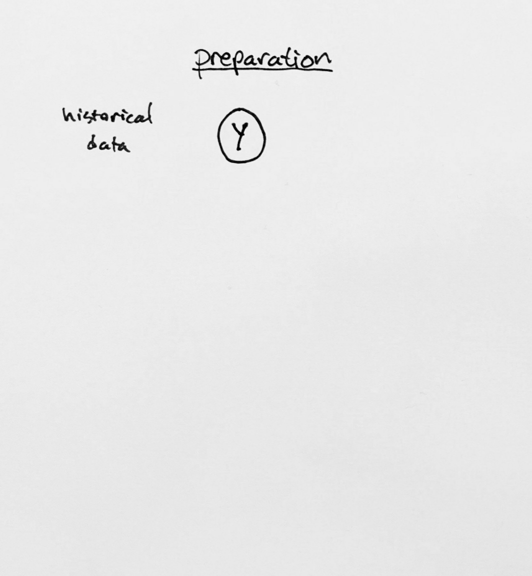
Review of CDF and Inverse CDF



Review of CDF and Inverse CDF



Preparation



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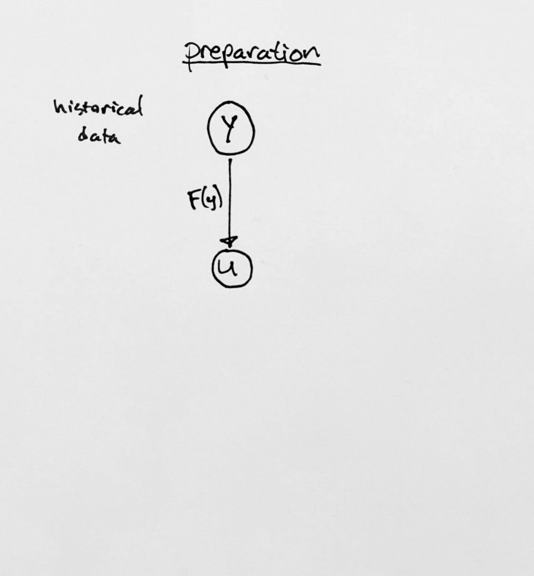

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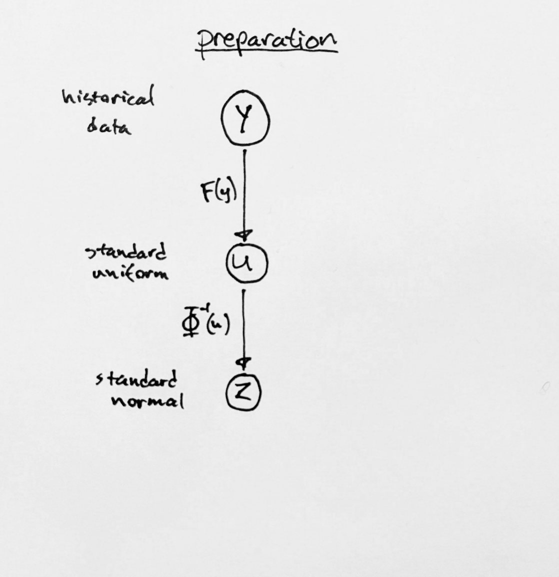

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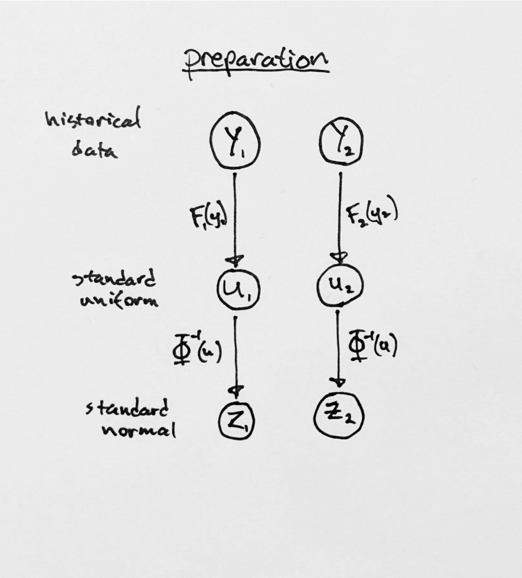

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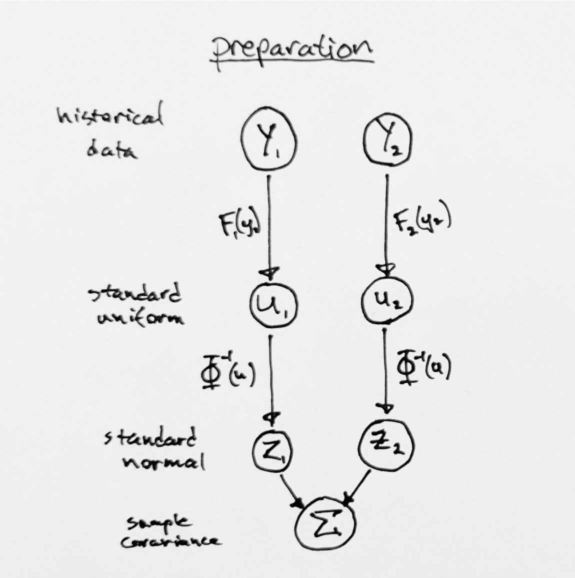
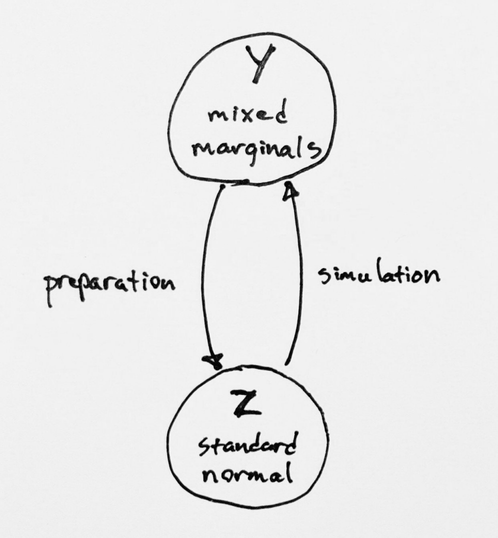


Preparation

- Marginal distributions for each variable Y_i are reflected in the individual F_i (equivelently F_i^{-1}), which were specified individually and separately from one another
- The dependence among the variables is captured in the sample covariance matrix (Σ) for the z_i

Overview


Y
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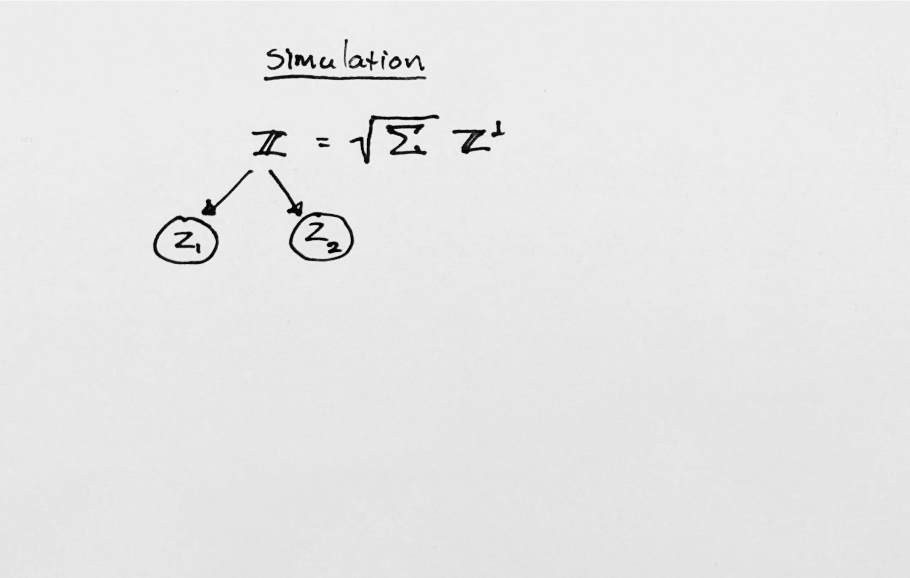
Simulation

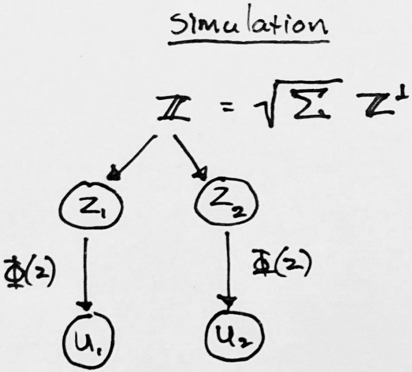
Simulation

$$\mathbb{X} = \sqrt{\Sigma}\, \mathbb{Z}^{\perp}$$

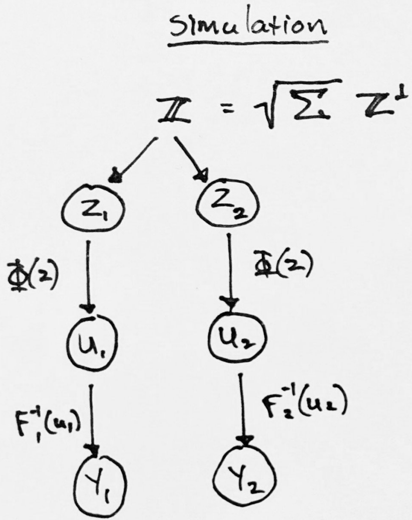
Simulation



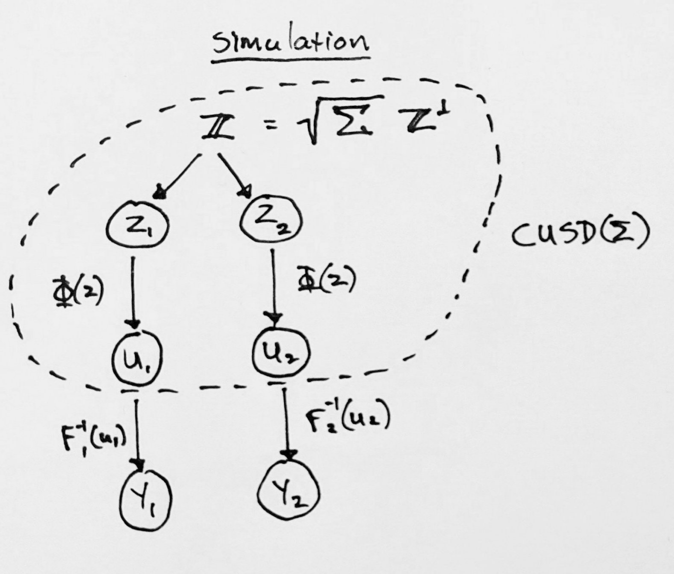
Simulation



Simulation



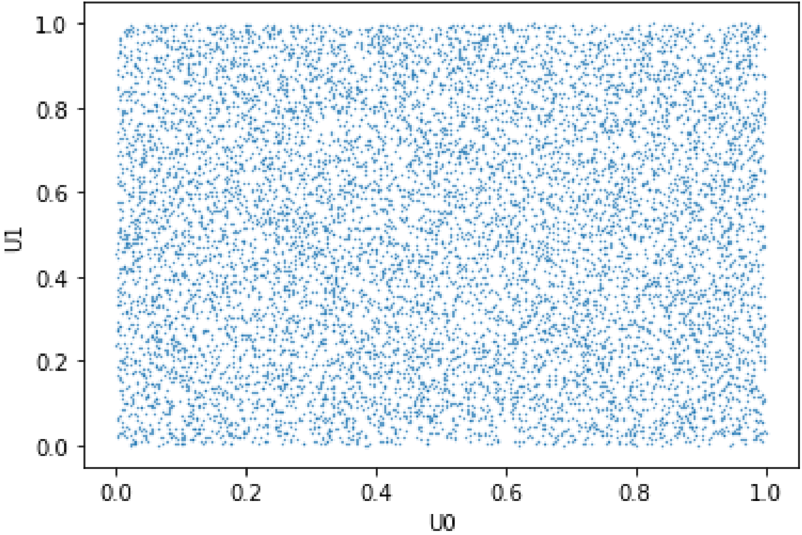
Simulation



Dependent U values

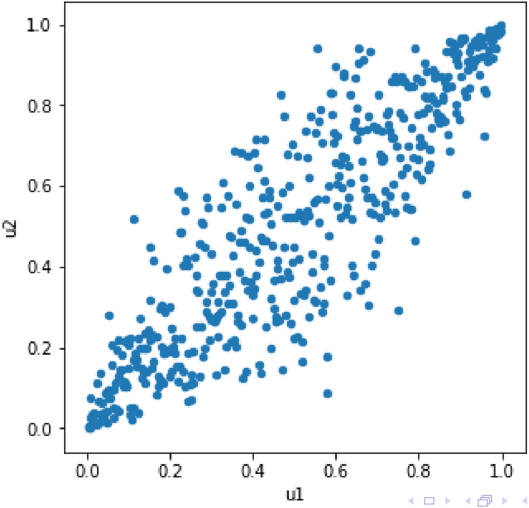
- The values we have generated using this process for the U variables reflect dependence among our Y variables

Independent Bivariate U Draws



Dependent Bivariate U Draws

Gaussian Copula, $\rho = 0.9$



Inverse CDFs Using *U* draws

- So far, we have mostly used Simetar's functions to generate stochastic draws without using the arguments for specifying *U* values
- Invisibly in the background, Simetar generated the *U* values automatically and *independently for each variable*
- To implement the last step in the simulation of mixed marginals, we will need to manually pass our *non-independent U* values to the inverse CDF functions

Inverse CDFs Using U draws

- NORM(μ, σ, u)
- UNIFORM(min, max, u)
- BETAINV(u, α, β, min, max)
- EMPIRICAL(historical sample, , u)

Empirical CDF

- So far, we used Simetar's EMPIRICAL function to generate a stochastic draw (a y value) using a u value (either implicitly or, now, explicitly)
- That is, we have been applying $F^{-1}(u)$ for the empirical distribution
- In the preparation phase of a mixed marginals analysis, (if we specify an empirical distribution for one or more of our variables), we need to apply $F(y)$
- This can *almost* be accomplished using an Excel function: $u =$ PERCENTRANK(historical sample, y)
- Unfortunately this will generate values of exactly one and zero that cannot be used by the inverse standard normal CDF
- Instead, use

$$MAX(0.001, MIN(0.999, PERCENTRANK(\text{historical sample}, y)))$$