Agribusiness Analysis and Forecasting

Autoregressive Process, Part II

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Time Series Model Estimation

Outline:

- Review stationarity and no. of lags.
- Discuss model estimation.
- Demonstrate how to estimate Time Series (AR) models with *Simetar*.
- Interpretation of model results.
- How to forecast the results for an AR model.

Time Series Model Estimation - Stationarity

- Plot the data to see the characteristics of the series you are analyzing.
- Use the Dickey-Fuller test to determine the minimum number of differences (possible zero) needed to render the data stationary (DF function in Simetar)
- Specify the number of lags in the AR model
  - Sample autocorrelations (AUTOCORR in Simetar)
  - Partial autocorrelations (PAUTOCORR)
  - Schwarz Information Criterion (ARLAG)

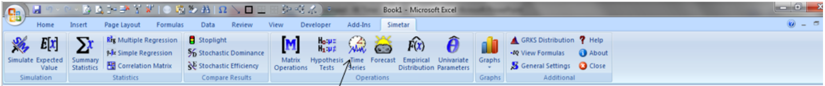
Time Series Model Estimation

- Once you have determined the number of differences in the AR model…
- Use OLS for estimation
- For a series requiring one difference and three lags, estimate

$$D_{1,t} = \beta_0 + \beta_1 D_{1,t-1} + \beta_2 D_{1,t-2} + \beta_3 D_{1,t-3}$$

- Use this equation to forecast the $D_{1,t+i}$ which implies forecasts for Y_{t+i}.

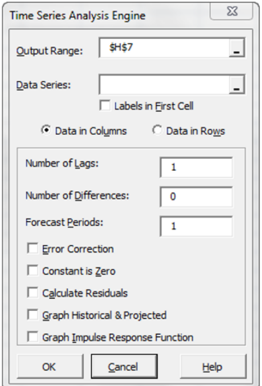
Time Series Model Estimation in Simetar



- An alternative to estimating the OLS regression model and having to forecast the model by hand

  we let Simetar do the work

- Simetar time series function is driven by a menu



Time Series Model Estimation

- Top line contains the fitted OLS coefficients.
- S.E. of Coef. can be used to calculate t ratios to infer which lags are significant.
- Can explore restricting out lags (variables).
- The SIC updates as restrictions are imposed, and can therefore be used to infer the appropriateness of restrictions

AR Series Analysis Results for 2 Lags & 1 Difference,							
	Constant	SalesL1	SalesL2				
Sales	3.393	0.476	-0.107				
S.E. of Coefficients							
Sales	30.764	0.144	0.143				
Restriction Matrix							
Sales	1	1	1				
Differences	1						
Characteristic	Dickey-Fu	Aug. Dick	Schwarz	S.D. Resi	MAPE	AIC	SIC
Sales	-4.471	-4.271	5.529	212.955	8.86	10.84	10.95345

Forecasting a Time Series Model

- If the original series is stationary and has T observations of data we estimate the model as an AR(0 differences, 1 lag)
- Forecast the first period ahead as:

$$\hat{Y}_{T+1} = \alpha + \beta Y_T$$

- Forecast the second period ahead as:

$$\hat{Y}_{T+2} = \alpha + \beta \hat{Y}_{T+1}$$

- Repeat for additional periods.
- This <u>ONLY</u> works if Y is stationary and the AR model reflects zero differences.

Forecasting a D_1 Times Series Model

- What if Y was non-stationary, and $D_{1,t}$ was stationary? Also, assume a single lag is appropriate for the AR model. How do you forecast?
- Let T represent the last historical observation.
- Steps for the one-period-ahead forecast:
  Recall that $D_{1,T} = Y_T - Y_{T-1}$.
- So the AR model projection is:

$$\hat{D}_{1,T+1} = \alpha + \beta D_{1,T}$$

- Next add the forecasted $\hat{D}_{1,T+1}$ to Y_T to forecast $\hat{Y}_{T+1}$ as follows:

$$\hat{Y}_{T+1} = Y_T + \hat{D}_{1,T+1}$$

.

Forecasting a D_1 Time Series Model

- Two-period-ahead forecast is:

$$\hat{D}_{1,T+2} = \alpha + \beta \hat{D}_{1,T+1}$$

$$\hat{Y}_{T+2} = \hat{Y}_{T+1} + \hat{D}_{1,T+2}$$

- Repeat the process for period 3 and so on.
- This is *recursive dynamic* forecasting

For Forecast Model $D_{1,t} = 4.019 + 0.42859\ D_{1,T-1}$

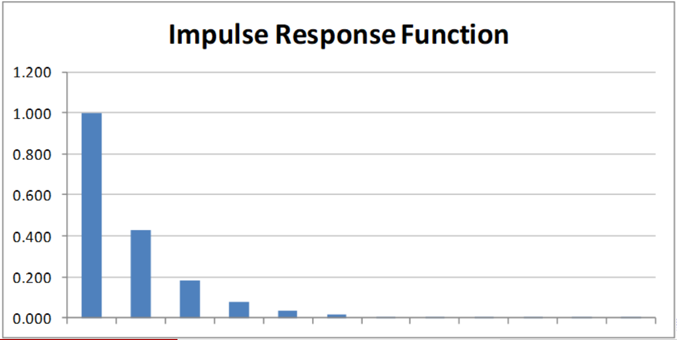
Year	History and Forecast $\hat{Y}_{T+i}$	Change $\hat{Y}$ or $\hat{D}_{1,T}$	Forecast D_{1T+i}	Forecast $\hat{Y}_{T+i}$
T-1	1387			
T	1289	-98.0	-37.925 = 4.019 + 0.428*(-98)	**1251.1** = 1289 + (-37.925)
T+1	**1251.1**	-37.9	-12.224 = 4.019 + 0.428*(-37.9)	**1238.91** = 1251.11 + (-12.22)
T+2	**1238.91**	-12.19	-1.198 = 4.019 + 0.428*(-12.19)	**1237.71**=**1238.91** + (-1.198)
T+3	**1237.71**			

Time Series Model Forecast-Note that this Model Restricted Out the Second Lag

AR Series Analysis Results for 2 Lags & 1 Difference						
	Constant	SalesL1	SalesL2			
Sales	3.028	0.430	0.000			
S.E. of Coefficients						
Sales	30.621	0.129	0.000			
Restriction Matrix						
Sales	1	1	0			
Differences	1					
Characteristic	Dickey-Fu	Aug. Dick	Schwarz	S.D. Resi	MAPE	AIC
Sales	-4.471	-4.271	5.529	214.1866	9.06	10.81
Forecast	Impulse	Auto-	t-Statistic	Partial	t-Statistic	
	Response	Correlatic	(AutoCorr	AutoCorre	(Part.Autc	Period
1,249.849	1.000	0.427042	3.108914	0.427042	3.108914	1
1,236.027	0.431	0.096647	0.602284	-0.10484	-0.76322	2
1,233.106	0.186	0.073858	0.457153	0.09031	0.657466	3
1,234.877	0.080	0.109992	0.678136	0.062501	0.455016	4
1,238.667	0.034	0.033193	0.202893	-0.05185	-0.37748	5

Time Series Model Estimation

- The *Impulse Response Function* shows the impact of a unit shock (in Simetar) in Y (or D_1) on the projected values of over time.
- The rate at which the shock dies out depends on the amount of persistence in the series (whcih should be reflected in a properly specified model)



Simulation of a Time Series Model

Stochastic recursive dynamic forecasts for an *AR* model.
Recall that the estimation was via OLS, and there exists an error component.

To make the recursive dynamic projections stochastic, add a stochastic error (or *innovation*) to the projection:

$$D_{1,T+1} = \alpha + \beta D_{1,T} + \epsilon_{T+1}$$

where (for now), assume ϵ is normally distributed with a mean of zero and a standard deviation estimated from the OLS residuals

Vector Autoregressive (VAR) Models

- VAR models are time series models where two or more variables are thought to be correlated and together they explain more than each variable by itself.
- For example forecasting
  - Sales and Advertising
  - Money supply and interest rate
  - Supply and Price
  - Corn price and soybean price

VAR Time Series Model Estimation

An example of advertising and sales

$$DA_{T+i} = \alpha + \beta_1 DA_{1,T-1} + \beta_2 DA_{1,T-2} + \gamma_1 DS_{1,T-1} + \gamma_2 DS_{1,T-2}$$
$$DS_{T+i} = \alpha + \beta_1 DS_{1,T-1} + \beta_2 DS_{1,T-2} + \gamma_1 DA_{1,T-1} + \gamma_2 DA_{1,T-2}$$

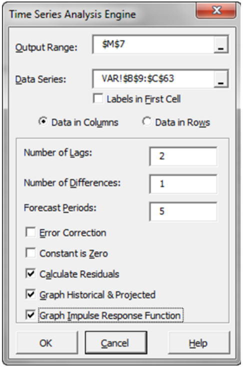
where

- A is advertising and S is sales
- DA is the difference for A and
- DS is the difference for S

In this model we fit A and S at the same time and A is affected by its lag differences and the lagged differences for S. The same is true for S affected by its own lags and those of A.

VAR Model Estimation

- Advertising and sales VAR model.
- Highlight two columns B and C
- Specify number of lags (applies to both series)
- Specify number differences (applies to both series)



VAR Model Estimation

Advertising and sales VAR model

VAR Series Analysis Results for 6 Lags & 1 Difference,

	Constant	Adv.L1	Adv.L2	Adv.L3	Adv.L4	Adv.L5	Adv.L6	SalesL1	SalesL2	SalesL3	SalesL4	SalesL5	SalesL6
Adv.	-5.049	-0.269	-0.499	-0.294	0.059	-0.252	-0.040	0.530	-0.074	0.205	0.089	0.263	-0.020
Sales	-1.614	0.003	-0.350	-0.343	-0.168	-0.555	-0.268	0.417	0.055	0.166	0.108	0.267	0.081
S.E. of Coefficients													
Adv.	27.243	0.189	0.192	0.205	0.197	0.182	0.186	0.153	0.175	0.171	0.175	0.173	0.165
Sales	32.920	0.228	0.232	0.248	0.238	0.220	0.225	0.185	0.212	0.207	0.211	0.210	0.199
Restriction Matrix													
Adv.	1	1	1	1	1	1	1	1	1	1	1	1	1
Sales	1	1	1	1	1	1	1	1	1	1	1	1	1
Differences	1												
Characteristi	Dickey-Fuller T	Aug. Dicke	LRT	LRT Critic	S.D. Resic	MAPE							
Sales	-6.744	-7.435	5.476	5.892	158.03	15.11							
	-4.471	-4.271			190.96	7.54							

Forecast		Impulse Response		
Adv.	Sales	Adv.	Sales	Period
523.275	1,256.382	1.000	1.000	1
517.318	1,256.811	-0.268	0.419	2
529.415	1,304.185	-0.425	-0.119	3
547.782	1,352.748	-0.231	-0.349	4
544.850	1,391.891	0.229	-0.054	5
561.290	1,434.949	-0.190	-0.026	6
586.582	1,471.055	-0.312	-0.381	7
606.375	1,498.413	-0.077	-0.225	8
609.176	1,511.007	0.166	0.189	9
612.212	1,519.553	-0.018	0.091	10