

Agribusiness Analysis and Forecasting

Seasonality and Cycles

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Uses for Seasonal Models

A **seasonal** pattern is a recurring pattern of variability in a time series that occurs of the course of each year.

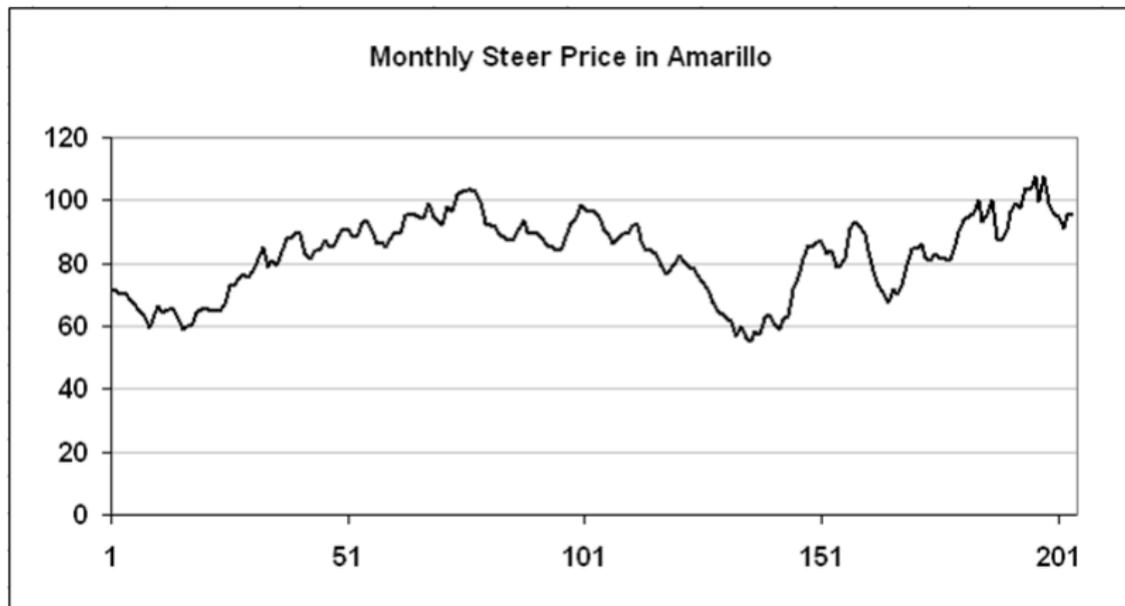
- Prices usually differ from one season to another.
 - Tomatoes, avocados, grapes, lettuce
 - Wheat, corn, hay
 - 450-550 pound Steers
 - Gasoline
- Many quantity series have seasonal patterns (e.g., production)
- To forecast seasonal data you must explicitly incorporate variables in the model to reflect the seasonality.

Seasonal Forecasts

- Multiple observations per year are needed to observe a seasonal pattern.
- Seasonal patterns repeat each year due to
 - Seasonal production due to climate or weather.
 - Seasonal demand (holidays, summer, etc.).
- A cycle may also be present, with a seasonal pattern mapped on the top of the cycle.

Seasonal Forecasts

Example of monthly prices showing seasonal variability on top of a multi-year cycle.



Econometric Models for Forecasting Seasonal Patterns

- Seasonal indices
- Composite forecast models
- Harmonic regression model
- Dummy variable regression model

Steps for Estimating a Seasonal Index

- Graph the data.
- Check for a trend and seasonal pattern.
- Develop and use a seasonal index if no trend is present.
- Develop a composite forecast model that includes trend and seasonal components.

Two kinds of Seasonal Indices

- Price Index
 - The traditional index value shows the relative relationship of price between months or quarters.
 - It is **ONLY** used with price data.
- Fractional Contribution Index
 - If the variable is a quantity, calculate a fractional contribution index to show the relative contribution of each month to the annual total quantity.
 - It is **ONLY** used with quantities.

Seasonal Price Index Model

- Seasonal price index is a simple way to forecast a monthly or quarterly series.
- Index represents the fraction that each month's price is above or below the annual mean.
 - If the seasonal index for June is 1.10 it means the June price averages 110% of the annual average price.
 - If the seasonal index for December is 0.85 it means the December price averages 85% of the annual average price.
- A function in Simetar estimates the seasonal price index for time series with a few simple steps, so it is very easy to use this type of forecasting model.

Seasonal Price Index Model Estimated by Simetar

| Years | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 71.06 | 71.47 | 70.06 | 70.31 | 68.75 | 67.08 | 64.8 | 63.12 | 59.44 | 63.94 | 66.62 | 64.12 |
| 2 | 65.12 | 65.25 | 62.72 | 59.15 | 60.19 | 60 | 64.5 | 65.25 | 66.13 | 64.7 | 64.94 | 64.68 |
| 3 | 66.88 | 72.6 | 73.66 | 75.43 | 76.38 | 75.63 | 77.53 | 81.5 | 85.4 | 78.81 | 80.71 | 79.5 |
| 4 | 83.66 | 88 | 88.3 | 89.75 | 89.5 | 82.8 | 81.5 | 84.1 | 84.25 | 87.5 | 85.7 | 85.33 |
| 5 | 89.13 | 90.88 | 90.5 | 88.25 | 88.4 | 92.83 | 93.83 | 90.7 | 86.5 | 87 | 85.1 | 88.08 |
| 6 | 89.85 | 89.63 | 95.13 | 95.25 | 95.8 | 94.63 | 94.38 | 99.2 | 94.75 | 93.3 | 91.88 | 98.17 |
| 7 | 96.25 | 101.75 | 102.75 | 103.3 | 103.19 | 102.69 | 99.63 | 92.94 | 92.19 | 91.85 | 89.38 | 88.25 |
| 8 | 87.44 | 87.69 | 91.15 | 93.88 | 90 | 89.4 | 89.25 | 88.01 | 85.75 | 85.44 | 84.25 | 84.13 |
| 9 | 88.63 | 92.88 | 94.35 | 98.32 | 97.44 | 96.45 | 96.34 | 95.07 | 90.5 | 88.82 | 86.5 | 87.67 |
| 10 | 89.57 | 89.5 | 92.4 | 91.88 | 87.55 | 84 | 84.34 | 83.1 | 79.32 | 76.57 | 77.85 | 80.08 |
| 11 | 82.45 | 80.51 | 78.88 | 78.19 | 75.9 | 73.87 | 71.83 | 67.4 | 65 | 63.5 | 62.5 | 61.5 |
| 12 | 56.9 | 60.07 | 56.49 | 54.94 | 58.3 | 57.28 | 62.67 | 63.94 | 60.47 | 59.14 | 62.31 | 63.01 |
| 13 | 71.99 | 75.8 | 81.49 | 85.48 | 85.15 | 86.6 | 86.63 | 82.98 | 84 | 78.79 | 79.14 | 81.32 |
| 14 | 90.83 | 93.17 | 91.86 | 89.43 | 83.85 | 77.815 | 72.92 | 70.915 | 67.275 | 71.63 | 70.445 | 72.835 |
| 15 | 79.465 | 84.82 | 84.405 | 86.25 | 81.755 | 81.16 | 83.04 | 81.215 | 81.52 | 80.805 | 84.18 | 90.25 |
| 16 | 93.675 | 94.99 | 96.125 | 100.36 | 93.265 | 95.245 | 100 | 87.925 | 87.22 | 90.31 | 96.63 | 98.975 |
| 17 | 97.72 | 103.825 | 103.47 | 107.545 | 99.585 | 107.5 | 99 | 96 | 95 | 91.16 | 95.135 | 96.14 |
| SUM | 1400.62 | 1442.835 | 1453.74 | 1467.715 | 1435.005 | 1424.98 | 1422.19 | 1393.365 | 1364.715 | 1353.265 | 1363.27 | 1384.04 |
| AVERAGE | 82.38941 | 84.87265 | 85.51412 | 86.33618 | 84.41206 | 83.82235 | 83.65824 | 81.96265 | 80.27735 | 79.60382 | 80.19235 | 81.41412 |
| ST DEV | 11.57589 | 11.9041 | 12.96476 | 14.22 | 12.60769 | 13.73284 | 12.45774 | 11.46838 | 11.57323 | 10.96217 | 10.82638 | 11.95711 |
| INDEX | 0.994 | 1.024 | 1.032 | 1.042 | 1.019 | 1.011 | 1.009 | 0.989 | 0.969 | 0.961 | 0.968 | 0.982 |
| FRAC. CONT. INDEX | 0.083 | 0.085 | 0.086 | 0.087 | 0.085 | 0.084 | 0.084 | 0.082 | 0.081 | 0.080 | 0.081 | 0.082 |
| INDEX LCI | 0.719 | 0.749 | 0.735 | 0.719 | 0.726 | 0.690 | 0.718 | 0.715 | 0.686 | 0.691 | 0.703 | 0.695 |
| INDEX UCI | 1.270 | 1.299 | 1.329 | 1.365 | 1.311 | 1.333 | 1.301 | 1.263 | 1.251 | 1.230 | 1.232 | 1.270 |
| STOCHASTIC INDICES | 0.944 | 0.989 | 1.023 | 1.110 | 1.034 | 1.033 | 0.991 | 0.938 | 0.999 | 0.928 | 0.944 | 0.969 |
| STOCHASTIC FRACTIONAL | 0.083 | 0.089 | 0.088 | 0.083 | 0.083 | 0.086 | 0.084 | 0.081 | 0.080 | 0.077 | 0.081 | 0.086 |
| ADJ. STOCH. INDICES | 0.952 | 0.997 | 1.031 | 1.119 | 1.042 | 1.041 | 0.999 | 0.946 | 1.008 | 0.935 | 0.952 | 0.977 |
| ADJ. STOCH. FRACTIONAL | 0.083 | 0.089 | 0.087 | 0.083 | 0.083 | 0.086 | 0.084 | 0.081 | 0.080 | 0.077 | 0.081 | 0.086 |

Using a Seasonal Price Index for Forecasting

- Seasonal index has an average of 1.0
- Use seasonal index to forecast monthly prices from annual average price forecast

$$P_{Jan} = \text{Annual Average Price} * Index_{Jan}$$

$$P_{Mar} = \text{Annual Average Price} * Index_{Mar}$$

- Suppose you have a trend forecasted annual average price of \$125, you can develop monthly forecasts using the seasonal price indices, as:

$$\text{January Price} = \$125 * 0.88 = \$110.0$$

$$\text{March Price} = \$125 * 1.08 = \$135.0$$

- The forecast can include risk by using a stochastic forecast of annual price.

Probabilistic Monthly Forecasts

We use the Adjusted Stochastic Indices at the bottom of the Simetar output

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-----------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 71.06 | 71.47 | 70.06 | 70.31 | 68.75 | 67.08 | 64.8 | 63.12 | 59.44 | 63.94 | 66.62 | 64.12 |
| 2 | 65.12 | 65.25 | 62.72 | 59.15 | 60.19 | 60 | 64.5 | 65.25 | 66.13 | 64.7 | 64.94 | 64.68 |
| 3 | 66.88 | 72.6 | 73.66 | 75.43 | 76.38 | 75.63 | 77.53 | 81.5 | 85.4 | 78.81 | 80.71 | 79.5 |
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| 6 | 89.85 | 89.63 | 95.13 | 95.25 | 95.8 | 94.63 | 94.38 | 99.2 | 94.75 | 93.3 | 91.88 | 98.17 |
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| 8 | 87.44 | 87.69 | 91.15 | 93.88 | 90 | 89.4 | 89.25 | 88.01 | 85.75 | 85.44 | 84.25 | 84.13 |
| 9 | 88.63 | 92.88 | 94.35 | 98.32 | 97.44 | 96.45 | 96.34 | 95.07 | 90.5 | 88.82 | 86.5 | 87.67 |
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| 13 | 71.99 | 75.8 | 81.49 | 85.48 | 85.15 | 86.6 | 86.63 | 82.98 | 84 | 78.79 | 79.14 | 81.32 |
| 14 | 90.83 | 93.17 | 91.86 | 89.43 | 83.85 | 77.815 | 72.92 | 70.915 | 67.275 | 71.63 | 70.445 | 72.835 |
| 15 | 79.465 | 84.82 | 84.405 | 86.25 | 81.755 | 81.16 | 83.04 | 81.215 | 81.52 | 80.805 | 84.18 | 90.25 |
| 16 | 93.675 | 94.99 | 96.125 | 100.36 | 93.265 | 95.245 | 100 | 87.925 | 87.22 | 90.31 | 96.63 | 98.975 |
| 17 | 97.72 | 103.825 | 103.47 | 107.545 | 99.585 | 107.5 | 99 | 96 | 95 | 91.16 | 95.135 | 96.14 |
| SUM | 1400.62 | 1442.835 | 1453.74 | 1467.715 | 1435.005 | 1424.98 | 1422.19 | 1393.365 | 1364.715 | 1353.265 | 1363.27 | 1384.04 |
| AVERAGE | 82.38941 | 84.87265 | 85.51412 | 86.33618 | 84.41206 | 83.82235 | 83.65824 | 81.96265 | 80.27735 | 79.60382 | 80.19235 | 81.41412 |
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| INDEX | 0.994 | 1.024 | 1.032 | 1.042 | 1.019 | 1.011 | 1.009 | 0.989 | 0.969 | 0.961 | 0.968 | 0.982 |
| FRAC. CONT. INDEX | 0.083 | 0.085 | 0.086 | 0.087 | 0.085 | 0.084 | 0.084 | 0.082 | 0.081 | 0.080 | 0.081 | 0.082 |
| INDEX LCI | 0.719 | 0.749 | 0.735 | 0.719 | 0.726 | 0.690 | 0.718 | 0.715 | 0.686 | 0.691 | 0.703 | 0.695 |
| INDEX UCI | 1.270 | 1.299 | 1.329 | 1.365 | 1.311 | 1.333 | 1.301 | 1.263 | 1.251 | 1.230 | 1.232 | 1.270 |
| STOCHASTIC INDICES | 1.021 | 0.986 | 1.006 | 1.054 | 0.963 | 1.046 | 1.013 | 0.970 | 1.006 | 0.937 | 0.952 | 1.058 |
| STOCHASTIC FRACTIONAL IND | 0.081 | 0.084 | 0.088 | 0.083 | 0.083 | 0.085 | 0.083 | 0.083 | 0.084 | 0.082 | 0.080 | 0.085 |
| ADJ.STOCH.INDICES | 1.020 | 0.985 | 1.005 | 1.053 | 0.962 | 1.045 | 1.012 | 0.969 | 1.005 | 0.936 | 0.951 | 1.057 |
| ADJ.STOCH.FRACTIONAL INDIC | 0.081 | 0.084 | 0.088 | 0.083 | 0.083 | 0.085 | 0.083 | 0.083 | 0.084 | 0.082 | 0.080 | 0.085 |

Probabilistic Monthly Forecasts

- Develop probabilistic forecast of the annual price. In this case a trend forecast is used of the annual average prices and 18th period is forecasted.
- Use the stochastic indices to simulate stochastic monthly forecasts.

| | F | G | H | I | J | K | L | M | N | O | P | Q | R |
|----|-----------------------------------------------------------------------|----------------------------------|-------|------------|--------|------------|-------|------------|-------|------------|-------|------------|-------|
| 33 | | | | | | | | | | | | | |
| 34 | STOCHASTIC INDICES | 0.951 | 1.031 | 1.006 | 1.104 | 1.084 | 0.993 | 0.982 | 1.020 | 0.945 | 0.990 | 0.944 | 1.030 |
| 35 | STOCHASTIC FRACTIO | 0.082 | 0.082 | 0.085 | 0.084 | 0.085 | 0.087 | 0.088 | 0.081 | 0.081 | 0.082 | 0.076 | 0.084 |
| 36 | ADJ.STOCH.INDICES | 0.945 | 1.024 | 0.999 | 1.097 | 1.076 | 0.986 | 0.976 | 1.013 | 0.939 | 0.983 | 0.938 | 1.023 |
| 37 | ADJ.STOCH.FRACTION | 0.082 | 0.082 | 0.085 | 0.084 | 0.086 | 0.087 | 0.088 | 0.081 | 0.081 | 0.082 | 0.077 | 0.084 |
| 38 | | | | | | | | | | | | | |
| 39 | Simulate a Monthly Stochastic Price give a Stochastic Annual Forecast | | | | | | | | | | | | |
| 40 | Stochastic Annual Price | | | | | | | | | | | | |
| 41 | | 97.59264 =Y48+Z48*18+NORM(0,Y57) | | | | | | | | | | | |
| 42 | | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| 43 | | 92.23 | 99.94 | 97.50 | 107.06 | 105.06 | 96.26 | 95.22 | 98.84 | 91.63 | 95.94 | 91.54 | 99.88 |
| 44 | | =SGS41*G36 | | =SGS41*I36 | | =SGS41*K36 | | =SGS41*M36 | | =SGS41*O36 | | =SGS41*Q36 | |

Simetar Simulation Results for 500 Iterations.

| Variable | 'Price Inde |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Mean | 88.83 | 91.52 | 92.23 | 93.09 | 91.03 | 90.40 | 90.23 | 88.40 | 86.58 | 85.86 | 86.50 | 87.79 |
| StDev | 11.11 | 11.51 | 11.78 | 11.80 | 11.67 | 11.60 | 11.54 | 11.27 | 11.06 | 10.96 | 11.09 | 11.08 |
| CV | 12.50 | 12.58 | 12.78 | 12.67 | 12.82 | 12.83 | 12.79 | 12.75 | 12.77 | 12.77 | 12.82 | 12.62 |
| Min | 56.46 | 57.42 | 58.01 | 60.49 | 59.10 | 58.23 | 57.92 | 54.15 | 55.61 | 54.51 | 54.63 | 51.63 |
| Max | 117.61 | 123.98 | 124.45 | 131.95 | 125.98 | 129.61 | 123.03 | 127.56 | 121.12 | 121.55 | 120.66 | 119.62 |
| Iteration | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |

Seasonal Price Index Model Estimated by Simetar

[demo]

Seasonal Fractional Contribution Index

- Fractional Contribution Index sums to 1.0 to represent 100% of annual quantity of sales.
- Each month's value is the fraction of total sales that month.
- Use a trend or structural model to forecast annual sales.

$$Sales_{Jan} = \text{Total Annual Sales} * Index_{Jan}$$

$$Sales_{Jun} = \text{Total Annual Sales} * Index_{Jun}$$

- For an annual sales forecast at 340,000 units

$$Sales_{Jan} = 340,000 * 0.050 = 17,000.0$$

$$Sales_{Jun} = 340,000 * 0.076 = 25,840.0$$

- This forecast is useful for input procurement and inventory management.
- The forecast can include risk by using a stochastic forecast of annual sales.

Harmonic Regression for Seasonal Models

- Add sin and cos functions in OLS regression to isolate seasonal variation.
- Define a variable N to represent the number of observations per year
 - Ex.: Monthly observations: $N = 12$
 - Ex.: Weekly observations: $N = 52$
- Create the X Matrix for OLS regression.

X_1 indexes observations : $T = 1, 2, 3, 4, 5, \dots$

$$X_2 = \sin\left(\frac{2\pi T_t}{N}\right)$$

$$X_3 = \cos\left(\frac{2\pi T_t}{N}\right)$$

- Fit the regression equation as:

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 X_{3,t}$$

Harmonic Regression for Seasonal Models

This is what the X matrix looks like for a Harmonic Regression.

| | A | B | C | D | E |
|----|------------------------|-------|----------|-------------------------|-------------------------|
| 7 | | | Let SL = | 12 | |
| 8 | Formulas for Period 1: | | | =SIN(2*PI()*C10/\$D\$7) | =COS(2*PI()*C10/\$D\$7) |
| 9 | | Price | T | Sin(2PiT/SL) | Cos(2PiT/SL) |
| 10 | Jan | 71.06 | 1 | 0.5000000 | 0.8660254 |
| 11 | Feb | 71.47 | 2 | 0.8660254 | 0.5000000 |
| 12 | Mar | 70.06 | 3 | 1.0000000 | 0.0000000 |
| 13 | Apr | 70.31 | 4 | 0.8660254 | -0.5000000 |
| 14 | May | 68.75 | 5 | 0.5000000 | -0.8660254 |
| 15 | Jun | 67.08 | 6 | 0.0000000 | -1.0000000 |
| 16 | Jul | 64.8 | 7 | -0.5000000 | -0.8660254 |
| 17 | Aug | 63.12 | 8 | -0.8660254 | -0.5000000 |
| 18 | Sep | 59.44 | 9 | -1.0000000 | 0.0000000 |
| 19 | Oct | 63.94 | 10 | -0.8660254 | 0.5000000 |
| 20 | Nov | 66.62 | 11 | -0.5000000 | 0.8660254 |
| 21 | Dec | 64.12 | 12 | 0.0000000 | 1.0000000 |
| 22 | Jan | 65.12 | 13 | 0.5000000 | 0.8660254 |
| 23 | Feb | 65.25 | 14 | 0.8660254 | 0.5000000 |

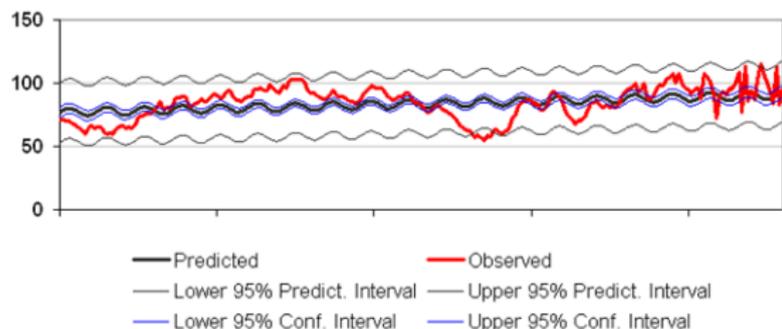
Harmonic Regression for Seasonal Models

- Note that the cos term is not statistically significant, but it MUST be included when using the model to forecast.
- sin and cos are creating the wave effect in the forecast.
- T is creating the positive trend in the forecast.
- The model needs more terms to capture the underlying cycle.

OLS Regression Statistics for Price.

| | | | | |
|-----------------------|---------------|------------|-------------|-------------------|
| F-test | 8.147 | Prob(F) | 0.000 | Unrestrict |
| MSE ^{1/2} | 11.835 | CV Regr | 14.281 | F-test |
| R ² | 0.109 | Durbin-W | 0.068 | R ² |
| RBar ² | 0.096 | Rho | 0.967 | RBar ² |
| Akaike Inf | 4.952 | Goldfeld-c | 0.360 | Akaike Inf |
| Schwarz I | 5.001 | | | Schwarz I |
| | 95% Intercept | T | Sin(2PiT/S) | Cos(2PiT/S) |
| Beta | 76.714 | 0.060 | 2.735 | -1.572 |
| S.E. | 1.665 | 0.014 | 1.173 | 1.172 |
| t-test | 46.083 | 4.265 | 2.331 | -1.342 |
| Prob(t) | 0.000 | 0.000 | 0.021 | 0.181 |
| Elasticity at Mean | | 0.074 | 0.000 | 0.000 |
| Variance Inflation Fc | | 1.002 | 1.002 | 1.000 |
| Partial Correlation | | 0.289 | 0.163 | -0.094 |
| Semipartial Correlat | | 0.284661 | 0.155613 | -0.08955 |

Observed and Predicted Values for Price



Recap of Harmonic Regression

$$Y_t = \beta_0 + \beta_1 Z_t + \beta_2 T_t + \beta_3 \sin\left(\frac{2\pi T_t}{N}\right) + \beta_4 \cos\left(\frac{2\pi T_t}{N}\right)$$

- The T variable captures the trend in the Y variable.
- The sin and cos capture the seasonal variability in Y .
- The Z variable represents structural variables that could explain changes due to income, population, tastes and preferences, policy shifts.

Harmonic Regression Demo

[demo]

Cycles

- Business cycle
- Beef cycle
- Hog cycle
- Earnings season

Analyzing and Forecasting Cycles

- Cyclical analysis involves analyzing data for underlying cycles.
- Estimate the length of an average cycle and forecast Y variable in part based on cycle length.
- May still include trend, seasonal, and structural variables to be removed with other parts of your model.
- Need adequate data to reflect several cycles

Rules for Cyclical Analysis Models

- Note that a cycle is simply a generalization of the seasonal concept
- Seasonal pattern: one cycle per year
- More general: cyclic pattern that repeats with an arbitrary frequency
- Now define N as the number of observations *per cycle* (previously number of obs per year)
- Example: to reflect a quarterly cycle with weekly obs, $N = 13$
- Example: to reflect a five-year cycle with monthly obs, $N = 60$

Cyclical Analysis Models

The regression model, including a possible trend, is still just

$$Y = \beta_0 + \beta_1 T + \beta_2 \sin\left(\frac{2\pi T}{N}\right) + \beta_3 \cos\left(\frac{2\pi T}{N}\right)$$

where N = number of observations per cycle.

Steps to estimate a cycle length:

- ① Enter N in a cell.
- ② Reference the cell with N to calculate all of the sin and cos values in the X matrix.
- ③ Estimate regression model.
- ④ Change the value for N , observe the F ratio, $MAPE$, R^2 , or information criterion
- ⑤ Repeat process for numerous N values and choose the N with the best value for your chosen fit metric

Multiple Simultaneous Cycles

OLS regression model:

$$Y = \beta_0 + \beta_1 T + \beta_2 \sin\left(\frac{2\pi T}{N_1}\right) + \beta_3 \cos\left(\frac{2\pi T}{N_1}\right) \\ + \beta_4 \sin\left(\frac{2\pi T}{N_2}\right) + \beta_5 \cos\left(\frac{2\pi T}{N_2}\right)$$

where: N_1 = Number of obs per cycle for the first cycle and N_2 = Number of obs per cycle for the second cycle

For example: make N_1 the number of observations per year to reflect a seasonal/annual cycle, and N_2 some larger number to reflect a multi-year cycle.

Results from a combined seasonal and multi-year cycle model

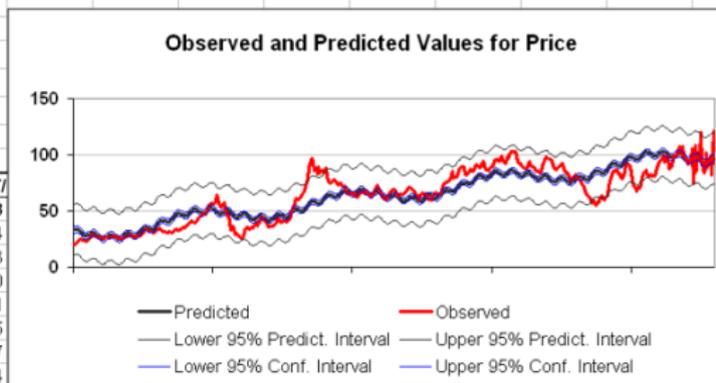
- Monthly data
- Slightly different notation than above: here "cycle length" is the of years (not obs) in the multi-year cycle, chosen based on maximum *MAPE*.

| | Test Alt. Cycles | | | | | | | |
|--------------|------------------|---------|--------|---------|----------------|--------|--------------|---------|
| Cycle length | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| R Squared | 0.752 | 0.748 | 0.759 | 0.748 | 0.78 | 0.811 | 0.813 | 0.798 |
| MAPE | 16.41 | 16.7239 | 16.551 | 16.3628 | 15.1482 | 15.221 | 15.8022 | 17.2846 |



OLS Regression Statistics for Price.

| | | | | | | | |
|-----------------------------------------------------------------|---------|------------|---------|---------------------------|---------|----------|--|
| F-test | 311.284 | Prob(F) | 0.000 | <u>Unrestricted Model</u> | | | |
| MSE ^{1/2} | 11.453 | CV Regr | 18.022 | F-test | 311.284 | | |
| R ² | 0.780 | Durbin-Wa | 0.057 | R ² | 0.780 | | |
| RBar ² | 0.778 | Rho | 0.970 | RBar ² | 0.778 | | |
| Akaike Inf | 4.886 | Goldfeld-C | 1.026 | Akaike Inf | 4.886 | | |
| Schwarz I | 4.932 | | | Schwarz I | 4.932 | | |
| 95% Intercept T Sin(2PIT)/S Cos(2PIT)/S Sin(2PIT)/C Cos(2PIT)/C | | | | | | | |
| Beta | 28.857 | 0.156 | 2.041 | -1.074 | -6.204 | 3.563 | |
| S.E. | 1.102 | 0.004 | 0.769 | 0.769 | 0.787 | 0.764 | |
| t-test | 26.180 | 36.162 | 2.655 | -1.398 | -7.887 | 4.663 | |
| Prob(t) | 0.000 | 0.000 | 0.008 | 0.163 | 0.000 | 0.000 | |
| Elasticity at Mean | | 0.546 | 0.000 | 0.000 | -0.001 | 0.001 | |
| Variance Inflation Fa | | 1.032 | 1.001 | 1.000 | 1.028 | 1.005 | |
| Partial Correlation | | 0.866 | 0.126 | -0.067 | -0.353 | 0.217 | |
| Semipartial Correlati | | 0.80973 | 0.05944 | -0.0313 | -0.1766 | 0.104414 | |



Cycle Demo

[demo]

Seasonal Forecast Using Dummy Variable Models

- Dummy variable regression model can forecast trend and seasonal variability.
- Include a trend if one is present.
- Regression model can be estimate as:

$$Y = \beta_0 + \beta_1 Jan + \beta_2 Feb + \dots + \beta_{11} Nov + \beta_{13} T + \beta_{14} Z$$

- Jan–Nov are individual dummy variable 0's and 1's.
- Effect of Dec is captured in the intercept.
- If the data are quarterly, use 3 dummy variables, for first 3 quarters and intercept picks up affect for fourth quarter.

$$Y = \beta_0 + \beta_1 Qt1 + \beta_2 Qt2 + \beta_3 Qt3 + \beta_4 T + \beta_5 Z$$

- The Z variable represents other structural variables that can be included in the model.

What is Each Part of the Equation Doing?

$$Y = \beta_0 + \beta_1 \text{Jan} + \beta_2 \text{Feb} + \dots + \beta_{11} \text{Nov} + \beta_{13} T + \beta_{14} Z$$

- The β_0 intercept is capturing the average value of Y when all the other variables are zero.
- The β_1 through β_{11} are capturing the effects of different months on the Y variable, some may be positive and others negative. Some may not be statistically significant but should be left in the model for completeness.
- The “T” variable is trend and effectively “de-trends” the data prior to estimating the seasonal effects.
- The “Z” variable represents other structural variables that can be included in the model, such as income, population, own price and competing prices.

Seasonal Forecast with Dummy Variable Models

- Set up the X matrix with 0's and 1's.
- Easy to forecast as the seasonal effect is assumed to persist forever.
- Note the pattern of 0s and 1s for months.
- December effect is captured in the intercept.

| | Price | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Trend |
|-----|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------|
| Jan | 71.06 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Feb | 71.47 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| Mar | 70.06 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| Apr | 70.31 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| May | 68.75 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 5 |
| Jun | 67.08 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 6 |
| Jul | 64.8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 7 |
| Aug | 63.12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 8 |
| Sep | 59.44 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 9 |
| Oct | 63.94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 10 |
| Nov | 66.62 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 11 |
| Dec | 64.12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12 |
| Jan | 65.12 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13 |
| Feb | 65.25 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 |
| Mar | 62.72 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15 |
| Apr | 59.15 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 |
| May | 60.19 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 17 |
| Jun | 60 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 18 |

Seasonal Forecast with Dummy Variable Models

- Regression output for a monthly dummy variable model may not have a statistically significant effect for each month, as indicated by the Student t on the betas.
- Monthly forecasts use beta for the month being forecasted.

$$\text{January forecast} = 45.93 + 4.147 * (1) + 1.553 * T$$

$$\text{May forecast} = 45.93 + 4.999 * (1) + 1.553 * T$$

| | 95% Intercept | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Trend |
|-----------------------|---------------|---------|----------|----------|----------|----------|----------|----------|----------|----------|----------|---------|----------|
| Beta | 45.930 | 4.147 | 6.334 | 6.682 | 7.213 | 4.999 | 4.121 | 3.671 | 1.689 | -0.281 | -1.240 | -0.936 | 1.553 |
| S.E. | 2.973 | 2.718 | 2.717 | 2.716 | 2.715 | 2.714 | 2.714 | 2.713 | 2.713 | 2.712 | 2.712 | 2.712 | 0.095 |
| t-test | 15.450 | 1.525 | 2.331 | 2.460 | 2.656 | 1.842 | 1.519 | 1.353 | 0.623 | -0.104 | -0.457 | -0.345 | 16.275 |
| Prob(t) | 0.000 | 0.129 | 0.021 | 0.015 | 0.009 | 0.067 | 0.131 | 0.178 | 0.534 | 0.918 | 0.648 | 0.730 | 0.000 |
| Elasticity at Mean | | 0.004 | 0.006 | 0.007 | 0.007 | 0.005 | 0.004 | 0.004 | 0.002 | 0.000 | -0.001 | -0.001 | 1.921 |
| Variance Inflation Fa | | 1.842 | 1.841 | 1.839 | 1.838 | 1.837 | 1.836 | 1.835 | 1.835 | 1.834 | 1.834 | 1.833 | 103.050 |
| Partial Correlation | | 0.110 | 0.167 | 0.176 | 0.190 | 0.133 | 0.110 | 0.098 | 0.045 | -0.008 | -0.033 | -0.025 | 0.764 |
| Semipartial Correlat | | 0.06802 | 0.103941 | 0.109692 | 0.118445 | 0.082118 | 0.067719 | 0.060325 | 0.027767 | -0.00463 | -0.02039 | -0.0154 | 0.725712 |
| Restriction | | | | | | | | | | | | | |