# Agribusiness Analysis and Forecasting

## Multiple Regression Forecasts

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# Multiple Regression Analysis

We would like to know following about regression analysis:

- How to use a regression to forecast a variable
- How to interpret the beta coefficients
- What the $t$ ratio means
- What the $p$ value is and what it means
- What the residuals are
- What the standard deviation is
- What is the $F$ ratio and $R^2$ and what they are used for

# Structural Variation

- Variables you want to forecast are often dependent on other variables.
  - Qt. Demand = f(Own Price, Substitute Price, Income, Population, Season, Tastes & Preferences, Trend, etc.).
  - $CropYield = a + b(Time, etc)$
- Structural models will explain most structural variation in a data series.
- Even when we build structural models, the forecast is not perfect.
- A residual remains as the unexplained portion.

# Multiple Regression Forecasts

- Variables to include in a structural model are suggested by:
  - Economic theory
  - Knowledge of industry
  - Known relationships to other variables

- Examples of forecasting and uses:
  - Planted acres – needed by ag. input businesses
  - Demand for a product - sales and processors
  - Price of corn for cattle - feedlots, grain mills, etc.
  - Government payments - Congressional Budget Office
  - Exports or trade flows - international ag. business

# Multiple Regression Forecasts

- Structural model

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + e$$

where $X$ and $Z$ are exogenous variables that explain the variation of $Y$ over the historical period.

- Estimate parameters $(\beta_0, \beta_1, \beta_2, e)$ using OLS.
  - OLS minimizes the sum of squared residuals.
  - That is, we seek to explain as much of the variation in $\hat{Y}$ as possible, i.e., maximizing the precision of your probabilistic forecast.

# Example of a Structural Forecast Model for a Crop

$$\text{Planted Acres}_t = f(Price_{t-1}, \text{Planted Acres}_{t-1}, IdleAcre_t, X_t)$$

$$HarvAc_t = f(PltAc_t)$$

$$Yield_t = f(Trend_t)$$

$$Production_t = Yield_t * HarvAc_t$$

$$Supply_t = Prod_t + EndStock_{t-1}$$

$$DomesticD_t = f(Price_t, Income/pop_t, Z_t)$$

$$Export_t = f(Price_t, Y_t)$$

$$EndingStock_t = f(Price_t, Production_t)$$

$$DomesticD_t + Export_t + Ending_Stock_t = Supply_t$$

# Steps to build Multiple Regression Models

- Plot the $Y$ variable in search of: trend, seasonal, cyclical, structural, and irregular variation.
- **Consider what relationships are suggested by economic theory and knowledge an industry**
- Plot $Y$ vs. each $X$ to evaluate the strength of hypothesized relationships; calculate correlation coefficients to $Y$.
- A Crop production forecast might be specified as follows:

$$PltAc_t = f(E(Price_t), PltAc_{t-1}, E(P^{th}Crop_t), Trend, Yield_{t-1})$$

$$HarvAc_t = \beta_0 + \beta_1 PltAc_t$$

$$Yield_t = \beta_0 + \beta_1 T$$

$$Prod_t = HarvAc_t * Yield_t$$

- Estimate with OLS.
- Make the deterministic forecast.
- Add stochastic error(s) for a probabilistic forecast.

# US Planted Wheat Acreage Model

$$PltAc_t = f(E(Price_t), Yield_{t-1}, CRP_t, Year_t)$$

| 95% | Intercept | Years | Price t-1 | CRP t | Yield t-1 |
|---|---|---|---|---|---|
| **Beta** | **843.985** | **-0.412** | **9.237** | -0.073 | 0.459 |
| **S.E.** | 350.136 | 0.183 | 1.297 | 0.138 | 0.412 |
| **t-test** | 2.410 | -2.245 | 7.124 | -0.529 | 1.113 |
| **Prob(t)** | 0.024 | 0.034 | 0.000 | 0.601 | 0.276 |
| **Elasticity at Mean** | | -11.508 | 0.407 | -0.008 | 0.229 |

- Statistically significant betas for the Trend ("Years") and Price.
- Leave *CRP* in model because its needed for policy analysis.
- Consider dropping $Yield_{t-1}$

# Forecasting with Multiple Regression Models

- Specify (assume) alternative values for $X$'s.
- Multiply Betas by their respective $X$'s.
- Forecast Acres for alternative Prices and *CRP*.
- Lagged Yield and Year are constant in scenarios.

| | C | D | E | F | G | H | I | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 92 | **Forecasting with alternative Values for Independent Variables** | | | | | | | | | | | |
| 93 | **Intercept** | **Years** | **Price t-1** | **CRP t** | **Yield t-1** | | | | | | | |
| 94 | **843.985** | **-0.412** | **9.237** | **-0.073** | **0.459** | **=M49** | | | | | | |
| 95 | **Assumed Values for the Independent Variables** | | | | | **Forecast Values** | | | | | | |
| 96 | Year | Price t-1 | CRP t | Yield t-1 | | Planted t | | | | | | |
| 97 | **2002** | **2.880** | **9.600** | **41.90** | | **64.996** | =$C$94+$D$94*C97+$E$94*D97+$F$94*E97+$G$94*F97 | | | | | |
| 98 | **2002** | **2.800** | **9.600** | **41.90** | | **64.257** | =$C$94+$D$94*C98+$E$94*D98+$F$94*E98+$G$94*F98 | | | | | |
| 99 | **2002** | **2.700** | **9.600** | **41.90** | | **63.334** | =$C$94+$D$94*C99+$E$94*D99+$F$94*E99+$G$94*F99 | | | | | |
| 100 | **2002** | **2.600** | **9.600** | **41.90** | | **62.410** | =$C$94+$D$94*C100+$E$94*D100+$F$94*E100+$G$94*F100 | | | | | |
| 101 | **2002** | **2.500** | **9.600** | **41.90** | | **61.486** | =$C$94+$D$94*C101+$E$94*D101+$F$94*E101+$G$94*F101 | | | | | |
| 102 | **2002** | **2.880** | **10.000** | **41.90** | | **64.967** | =$C$94+$D$94*C102+$E$94*D102+$F$94*E102+$G$94*F102 | | | | | |
| 103 | **2002** | **2.880** | **10.500** | **41.90** | | **64.930** | =$C$94+$D$94*C103+$E$94*D103+$F$94*E103+$G$94*F103 | | | | | |
| 104 | **2002** | **2.880** | **11.000** | **41.90** | | **64.894** | =$C$94+$D$94*C104+$E$94*D104+$F$94*E104+$G$94*F104 | | | | | |
| 105 | **2002** | **2.880** | **11.500** | **41.90** | | **64.857** | =$C$94+$D$94*C105+$E$94*D105+$F$94*E105+$G$94*F105 | | | | | |
| 106 | **2002** | **2.880** | **12.000** | **41.90** | | **64.821** | =$C$94+$D$94*C106+$E$94*D106+$F$94*E106+$G$94*F106 | | | | | |

# Multiple Regression Forecast with Risk

We will begin probabilistic forecast using $\tilde{PA}_{t+i}$ and $\sigma$ (Std. Dev) and assume a normal distribution for residuals.

$$\tilde{PA}_{t+i} = \hat{PA}_{t+i} + NORM(0, \sigma)$$

or

$$\tilde{PA}_{t+i} = \hat{PA}_{t+i} + \sigma * NORM(0, 1)$$

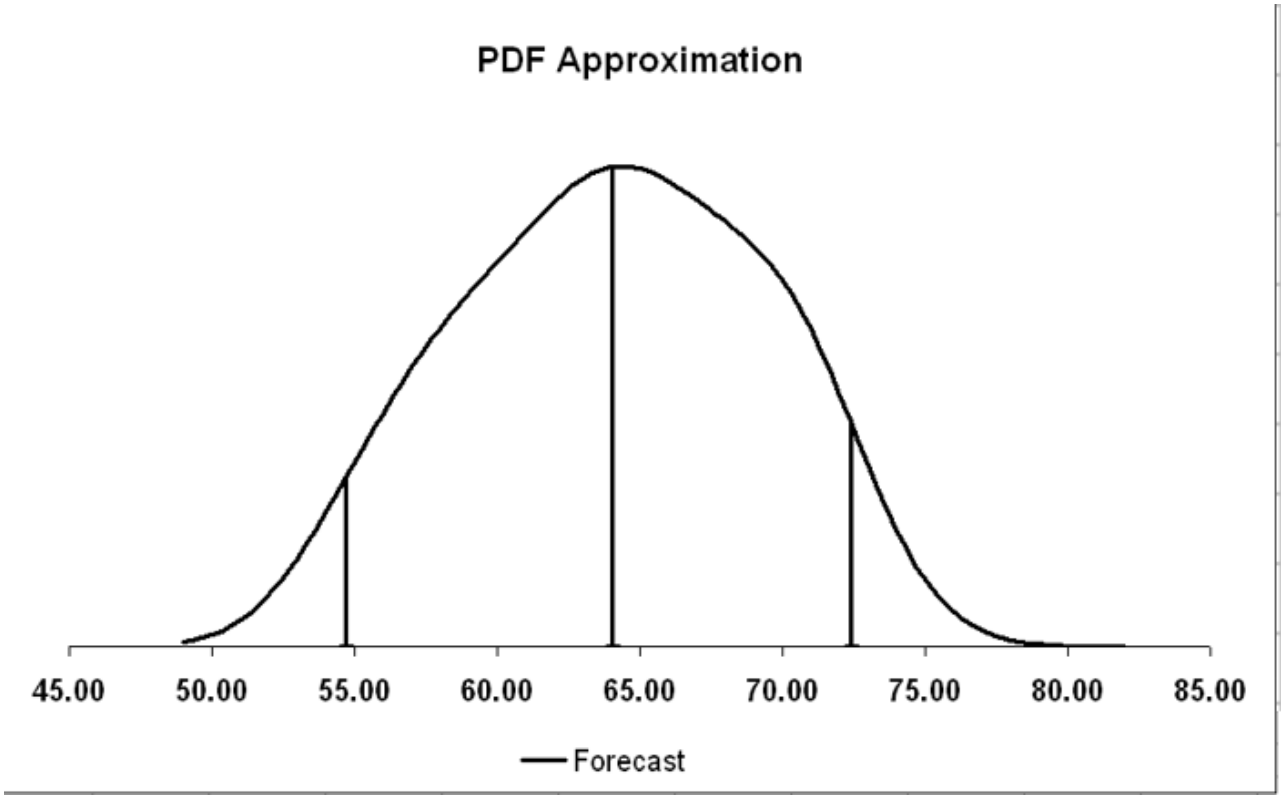
or

$$\tilde{PA}_{t+i} = NORM(\hat{PA}_{t+i}, \sigma)$$

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 108 | Intercept | Years | Price t-1 | CRP t | Yield t-1 | | |
| 109 | 843.985 | -0.41164 | 9.23694 | -0.07315 | 0.458552 | | |
| 110 | Assumed > | 2003 | 2.88 | 9.6 | 41.1 | | |
| 111 | Forecasted | | SE Predicted | | Probabilistic | | |
| 112 | Y-Hat for 2003 | | SEP for 2003 | | Forecast 2003 | | |
| 113 | 64.218 | | 5.141 | | 57.044 | =NORM(A113,C113) | |
| 114 | Formula to forecast determinisitc component | | | | | | |
| 115 | =A109+B109*B110+C109*C110+D109*D110+E109*E110 | | | | | | |

# Multiple Regression Forecast with Risk

Present probabilistic forecast as a PDF with 95% Confidence Interval shown here as the bars about the mean for a probability density function (PDF).

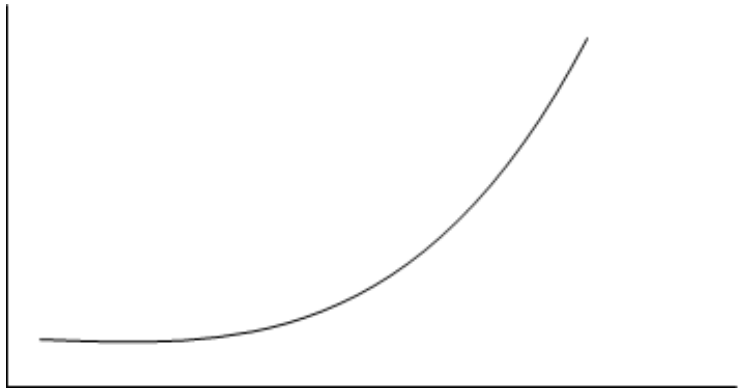



| PDF Approximation | |
|---|---|
| | Forecast 2 |
| Start | 49 |
| End | 82 |
| Band Widt | 2.157497 |
| Kernel | Gaussian |
| Confidenc | 95.0% |
| Lower Qua | 54.685 |
| Average | 64.045 |
| Upper Qua | 72.378 |

# Regression Model for Growth

- Some data display a growth pattern.
- Easy to forecast with multiple regression
- Add $T^2$ variable to capture the growth or decay of $Y$ variable.
- Growth function

$$Y = a + b_1 T + b_2 T^2$$



- Growth at decreasing rate

$$Y = a + b_1 Log(T)$$

# Data & Results for Log Models

## Single Log Form

$$Log(Y_t) = b_0 + b_1 T$$

| Try the Single Log Form | | | 95% | Intercept | Years |
|---|---|---|---|---|---|
| KOV | Log Y | Years | **Beta** | **184.039** | **-0.091** |
| 110.0 | 4.70048 | 1970 | **S.E.** | 4.393 | 0.002 |
| 93.0 | 4.532599 | 1971 | **t-test** | 41.895 | -41.176 |
| 80.0 | 4.382027 | 1972 | **Prob(t)** | 0.000 | 0.000 |
| 81.0 | 4.394449 | 1973 | **Elasticity at Mean** | | -57.184 |
| 68.0 | 4.219508 | 1974 | **Variance Inflation Factor** | | NA |
| 55.0 | 4.007333 | 1975 | **Partial Correlation** | | NA |
| 52.0 | 3.951244 | 1976 | **Semipartial Correlation** | | NA |

## Double Log Form

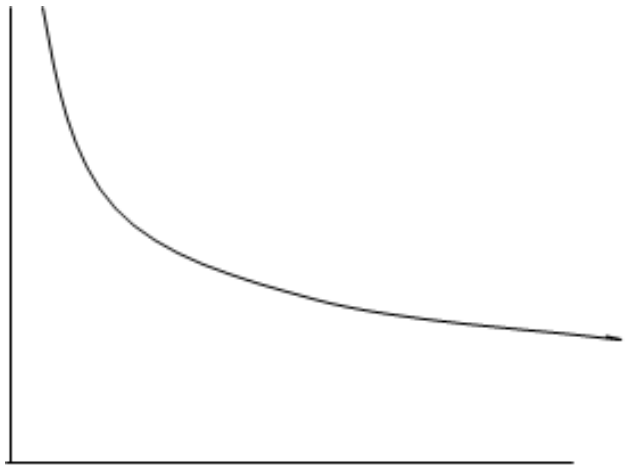
$$Log(Y_t) = b_0 + b_1 Log(T)$$

| Double Log Transformation of the Data | | | | 95% | Intercept | Log X |
|---|---|---|---|---|---|---|
| Actual KO | Years | Log Y | Log X | **Beta** | **1376.746** | **-180.887** |
| 110.0 | 1970 | 4.70048 | 7.585788822 | **S.E.** | 33.222 | 4.375 |
| 93.0 | 1971 | 4.532599 | 7.586296307 | **t-test** | 41.441 | -41.346 |
| 80.0 | 1972 | 4.382027 | 7.586803535 | **Prob(t)** | 0.000 | 0.000 |
| 81.0 | 1973 | 4.394449 | 7.587310506 | **Elasticity at Mean** | | -434.258 |

# Regression Model For Decay Functions

- Some data display a decay pattern.
- There are various possible models for this situation
- One example: use an exogenous variable of the form $\frac{1}{T}$
- Decay function

$$Y = \beta_0 + \beta_1(\frac{1}{T}) + \beta_2(\frac{1}{T^2})$$



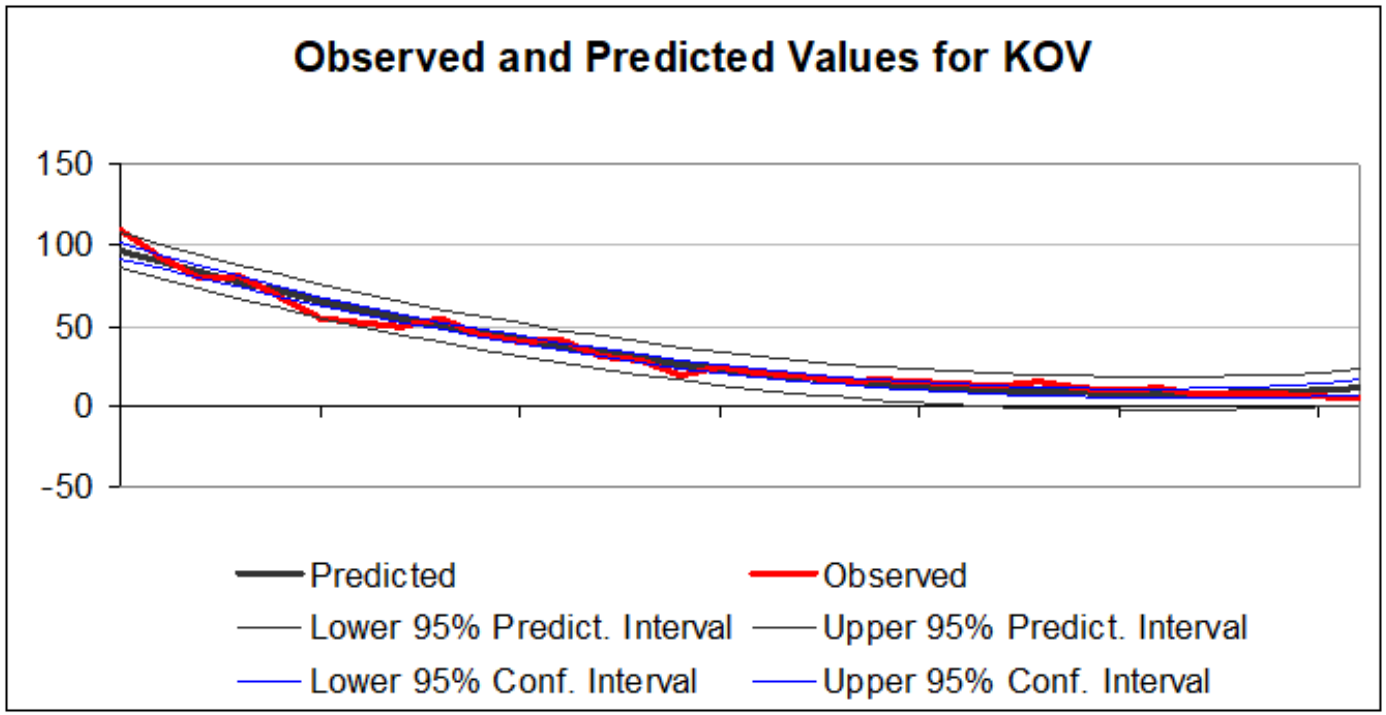
# Forecasting Growth or Decay Patterns

Here is the regression result for estimating a decay function.

$$Y_t = \beta_0 + \beta_1(\frac{1}{T})$$

or

$$Y_t = \beta_0 + \beta_1\frac{1}{T} + \beta_2\frac{1}{T^2}$$



# Multiple Regression Forecasts

Example of a structural regression model that contains both a *Trend* and an independent $X$ variable

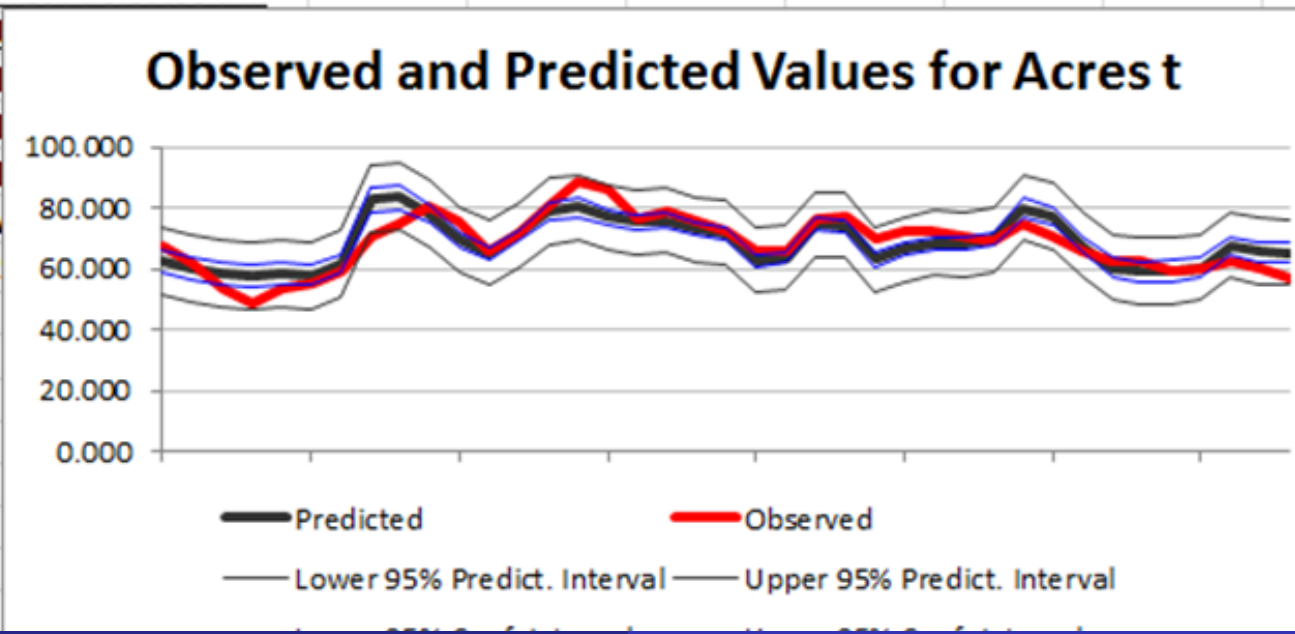
$$Y = \beta_0 + \beta_1 T + \beta_2 X_t$$

This equation does not explain all of the variability, a seasonal or cyclical variability may be present, if so, you need to "remove" its effect.

OLS Regression Statistics for Acres t

| | | | |
|---|---|---|---|
| F-test | 43.620 | Prob(F) | 0.000 |
| $MSE^{1/2}$ | 5.099 | CV Regr | 7.446 |
| $R^2$ | 0.708 | Durbin-W | 0.659 |
| $RBar^2$ | 0.692 | Rho | 0.671 |
| Akaike Inf | 3.281 | Goldfeld-( | 4.614 |
| Schwarz I | 3.366 | | |

| 95% | Intercept | EPrice t-1 | Trend |
|---|---|---|---|
| Beta | 805.020 | 9.758 | -0.386 |
| S.E. | 162.842 | 1.045 | 0.083 |
| t-test | 4.944 | 9.339 | -4.659 |
| Prob(t) | 0.000 | 0.000 | 0.000 |
| Elasticity at Mean | | 0.424 | -11.180 |
| Variance Inflation Factor | | | |
| Partial Correlation | | | |



# Regression evaluation

- Make sure parameter signs are based on sound economic theory for all variables.
- Student $t$ ratios greater than 1.96 ($P$ values for betas $< 0.05$).
- $F$ ratio larger than 20.0 and its $P$ value $< 0.05$.
- Add explanatory elements that increase $R^2$ by a non-trivial amount.

# Goodness of Fit Measures

Models with high $R^2$ may not forecast well. *Every* time we add a new $X$, we will get some increase in $R^2$.

$$R^2 = 1 - \frac{\sum_{t=1}^{T} e_t^2}{\sum_{t=1}^{T}(Y_t - \bar{Y})^2}$$

$\bar{R}^2$ is preferred as it is not affected by no. of $X$s.

$$\bar{R}^2 = 1 - \frac{(1 - R^2)(n-1)}{n-k}$$

We might also use MAPE for model selection. MAPE is used to measure the accuracy of a forecasting measure and usually the accuracy is expressed in terms of percentage.

$$MAPE = \frac{100\%}{n} \sum_{t=1}^{T} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|$$

# Information Criteria

For a residual sum of squares $RSS$, a number of right-hand-side variables $k$, and a number of observations $n$, Akaike Information Criterion and Bayesian Information Criterion are calculated as follows:

- Akaike Information Criterion (AIC)

$$AIC = \ln\left(\frac{RSS}{n}\right) + \frac{2(k+1)}{n}$$

- Bayesian Information Criterion (BIC), sometimes called Schwartz Information Criterion (SIC) or Schwarz Bayesian Criterion (SBC)

$$BIC = \ln\left(\frac{RSS}{n}\right) + \frac{k}{n}ln(n)$$

- When fitting models, it is possible to increase the likelihood by adding parameters, but doing so may result in overfitting. The AIC & BIC resolve this problem by introducing a penalty term for the number of parameters in the model. The penalty term is larger in BIC than in AIC.

# Summary of Goodness of Fit Measures

- *MAPE* works best to determine model for "in sample" forecasting.
- $R^2$ does not penalize for adding $X$s.
- $\bar{R}^2$ provides some penalty as $k$ increases.
- *AIC* is better than $R^2$ but *BIC* results in the most parsimonious models (fewest $X$s).

# Black Swans (BSs)

- Black Swans are low probability events.
  - An outlier, which is "outside realm of reasonable expectations."
  - Carries an extreme impact.
  - Human nature causes us to concoct explanations.
- Black swans are an example of uncertainty
  - Uncertainty is generated by unknown probability distributions.
  - Risk is generated by "known" distributions.
- 1917 influenza pandemic would be a black swan.
- War outbreak
- Meteor strike
- Sudden policy change
- We will discuss more about incorporating uncertainty in the model when we start simulating non-parametric distributions.