Agribusiness Analysis and Forecasting

Autoregressive Process, Part II

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Texas A&M University
Time Series Model Estimation

Outline:

- Review stationarity and no. of lags.
- Discuss model estimation.
- Demonstrate how to estimate Time Series (AR) models with Simetar.
- Interpretation of model results.
- How to forecast the results for an AR model.
Plot the data to see the characteristics of the series you are analyzing.

Use the Dickey-Fuller test to determine the minimum number of differences (possible zero) needed to render the data stationary (DF function in Simetar)

Specify the number of lags in the AR model
  - Sample autocorrelations (AUTOCORR in Simetar)
  - Partial autocorrelations (PAUTOCORR)
  - Schwarz Information Criterion (ARLAG)
Time Series Model Estimation

- Once you have determined the number of differences in the AR model...
- Use OLS for estimation
- For a series requiring one difference and three lags, estimate
  \[ D_{1,t} = \beta_0 + \beta_1 D_{1,t-1} + \beta_2 D_{1,t-2} + \beta_3 D_{1,t-3} \]
- Use this equation to forecast the \( D_{1,t+i} \) which implies forecasts for \( Y_{t+i} \).
Time Series Model Estimation in Simetar

• An alternative to estimating the OLS regression model and having to forecast the model by hand, we let Simetar do the work.

• Simetar time series function is driven by a menu.
Time Series Model Estimation

- Top line contains the fitted OLS coefficients.
- S.E. of Coef. can be used to calculate $t$ ratios to infer which lags are significant.
- Can explore restricting out lags (variables).
- The SIC updates as restrictions are imposed, and can therefore be used to infer the appropriateness of restrictions.

### AR Series Analysis Results for 2 Lags & 1 Difference.

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>SalesL1</th>
<th>SalesL2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>3.393</td>
<td>0.476</td>
<td>-0.107</td>
</tr>
<tr>
<td>S.E. of Coefficients</td>
<td>30.764</td>
<td>0.144</td>
<td>0.143</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Dickey-Fu Aug.</th>
<th>Dick Schwarz</th>
<th>S.D. Resi</th>
<th>MAPE</th>
<th>AIC</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>-4.471</td>
<td>-4.271</td>
<td>5.529</td>
<td>212.955</td>
<td>8.86</td>
<td>10.84</td>
</tr>
</tbody>
</table>
Forecasting a Time Series Model

- If the original series is stationary and has $T$ observations of data we estimate the model as an AR(0 differences, 1 lag)
- Forecast the first period ahead as:
  \[ \hat{Y}_{T+1} = \alpha + \beta Y_T \]
- Forecast the second period ahead as:
  \[ \hat{Y}_{T+2} = \alpha + \beta \hat{Y}_{T+1} \]
- Repeat for additional periods.
- This **ONLY** works if $Y$ is stationary and the AR model reflects zero differences.
Forecasting a $D_1$ Times Series Model

- What if $Y$ was non-stationary, and $D_{1,t}$ was stationary? Also, assume a single lag is appropriate for the AR model. How do you forecast?
- Let $T$ represent the last historical observation.
- Steps for the one-period-ahead forecast:
  Recall that $D_{1,T} = Y_T - Y_{T-1}$.
  So the AR model projection is:
  \[
  \hat{D}_{1,T+1} = \alpha + \beta D_{1,T}
  \]
  Next add the forecasted $\hat{D}_{1,T+1}$ to $Y_T$ to forecast $\hat{Y}_{T+1}$ as follows:
  \[
  \hat{Y}_{T+1} = Y_T + \hat{D}_{1,T+1}
  \]
Forecasting a $D_1$ Time Series Model

- Two-period-ahead forecast is:

  $$\hat{D}_{1,T+2} = \alpha + \beta \hat{D}_{1,T+1}$$

  $$\hat{Y}_{T+2} = \hat{Y}_{T+1} + \hat{D}_{1,T+2}$$

- Repeat the process for period 3 and so on.

- This is *recursive dynamic* forecasting
For Forecast Model $D_{1,t} = 4.019 + 0.42859 \, D_{1,T-1}$

<table>
<thead>
<tr>
<th>Year</th>
<th>History and Forecast $\hat{Y}_{T+i}$</th>
<th>Change $\hat{Y}$ or $D_{1,T}$</th>
<th>Forecast $D_{1T+i}$</th>
<th>Forecast $\hat{Y}_{T+i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1</td>
<td>1387</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>1289</td>
<td>-98.0</td>
<td>-37.925 = 4.019 + 0.428*(-98)</td>
<td>1251.1 = 1289 + (-37.925)</td>
</tr>
<tr>
<td>T+1</td>
<td><strong>1251.1</strong></td>
<td>-37.9</td>
<td>-12.224 = 4.019 + 0.428*(-37.9)</td>
<td><strong>1238.91</strong> = 1251.11 + (-12.22)</td>
</tr>
<tr>
<td>T+2</td>
<td><strong>1238.91</strong></td>
<td>-12.19</td>
<td>-1.198 = 4.019 + 0.428*(-12.19)</td>
<td><strong>1237.71</strong> = <strong>1238.91</strong> + (-1.198)</td>
</tr>
<tr>
<td>T+3</td>
<td><strong>1237.71</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Time Series Model Forecast—Note that this Model Restricted Out the Second Lag

<table>
<thead>
<tr>
<th>AR Series Analysis Results for 2 Lags &amp; 1 Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
</tr>
<tr>
<td>Sales</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S.E. of Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Restriction Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
</tr>
<tr>
<td>Differences</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Dickey-Fu</th>
<th>Aug. Dick</th>
<th>Schwarz</th>
<th>S.D. Resid</th>
<th>MAPE</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
<td>-4.471</td>
<td>-4.271</td>
<td>5.529</td>
<td>214.1866</td>
<td>9.06</td>
<td>10.81</td>
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</table>

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,249.849</td>
<td>1.000</td>
<td>0.427042</td>
<td>3.108914</td>
<td>0.427042</td>
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<tr>
<td>1,236.027</td>
<td>0.431</td>
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<td>-0.10484</td>
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<tr>
<td>1,233.106</td>
<td>0.186</td>
<td>0.073858</td>
<td>0.457153</td>
<td>0.09031</td>
</tr>
<tr>
<td>1,234.877</td>
<td>0.080</td>
<td>0.109992</td>
<td>0.678136</td>
<td>0.062501</td>
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<tr>
<td>1,238.667</td>
<td>0.034</td>
<td>0.033193</td>
<td>0.202893</td>
<td>-0.05185</td>
</tr>
</tbody>
</table>
Time Series Model Estimation

- The *Impulse Response Function* shows the impact of a unit shock (in Simetar) in \( Y \) (or \( D_1 \)) on the projected values of over time.
- The rate at which the shock dies out depends on the amount of persistence in the series (which should be reflected in a properly specified model).
Simulation of a Time Series Model

Stochastic recursive dynamic forecasts for an AR model. Recall that the estimation was via OLS, and there exists an error component.

To make the recursive dynamic projections stochastic, add a stochastic error (or *innovation*) to the projection:

\[ D_{1,T+1} = \alpha + \beta D_{1,T} + \epsilon_{T+1} \]

where (for now), assume \( \epsilon \) is normally distributed with a mean of zero and a standard deviation estimated from the OLS residuals.
Vector Autoregressive (VAR) Models

VAR models are time series models where two or more variables are thought to be correlated and together they explain more than each variable by itself.

For example forecasting
- Sales and Advertising
- Money supply and interest rate
- Supply and Price
- Corn price and soybean price
VAR Time Series Model Estimation

An example of advertising and sales

\[ DA_{T+i} = \alpha + \beta_1 DA_{1,T-1} + \beta_2 DA_{1,T-2} + \gamma_1 DS_{1,T-1} + \gamma_2 DS_{1,T-2} \]
\[ DS_{T+i} = \alpha + \beta_1 DS_{1,T-1} + \beta_2 DS_{1,T-2} + \gamma_1 DA_{1,T-1} + \gamma_2 DA_{1,T-2} \]

where

- \( A \) is advertising and \( S \) is sales
- \( DA \) is the difference for \( A \) and
- \( DS \) is the difference for \( S \)

In this model we fit \( A \) and \( S \) at the same time and \( A \) is affected by its lag differences and the lagged differences for \( S \). The same is true for \( S \) affected by its own lags and those of \( A \).
VAR Model Estimation

- Advertising and sales VAR model.
- Highlight two columns B and C
- Specify number of lags (applies to both series)
- Specify number differences (applies to both series)
VAR Model Estimation

Advertising and sales VAR model

VAR Series Analysis Results for 6 Lags & 1 Difference:

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Adv.</td>
<td>-5.049</td>
<td>-0.269</td>
<td>-0.499</td>
<td>-0.294</td>
<td>0.059</td>
<td>-0.252</td>
<td>-0.040</td>
<td>0.530</td>
<td>-0.074</td>
<td>0.205</td>
<td>0.089</td>
<td>0.263</td>
<td>-0.020</td>
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<tr>
<td>Sales</td>
<td>-1.614</td>
<td>0.003</td>
<td>-0.350</td>
<td>-0.343</td>
<td>-0.168</td>
<td>-0.555</td>
<td>-0.268</td>
<td>0.417</td>
<td>0.055</td>
<td>0.166</td>
<td>0.108</td>
<td>0.267</td>
<td>0.081</td>
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</table>

S.E. of Coefficients

<table>
<thead>
<tr>
<th></th>
<th>Adv.</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adv.</td>
<td>27.243</td>
<td>0.189</td>
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<tr>
<td>Sales</td>
<td>32.920</td>
<td>0.228</td>
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Restriction Matrix

<table>
<thead>
<tr>
<th></th>
<th>Adv.</th>
<th>Sales</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adv.</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Differences</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Characteristics:

<table>
<thead>
<tr>
<th></th>
<th>Dickey-Fuller T</th>
<th>Aug. Dickey LRT</th>
<th>LRT Critic</th>
<th>S.D. ResicMAPE</th>
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</thead>
<tbody>
<tr>
<td>Sales</td>
<td>-6.744</td>
<td>-7.435</td>
<td>5.476</td>
<td>5.892</td>
</tr>
<tr>
<td></td>
<td>-4.741</td>
<td>-4.271</td>
<td>158.03</td>
<td>15.11</td>
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Historical & Predicted

Forecast

<table>
<thead>
<tr>
<th>Adv. Sales</th>
<th>Impulse Response</th>
<th>Period</th>
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</thead>
<tbody>
<tr>
<td>523.275</td>
<td>1,256.382</td>
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</tr>
<tr>
<td>517.318</td>
<td>1,256.811</td>
<td>-0.268</td>
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<tr>
<td>529.415</td>
<td>1,304.185</td>
<td>-0.425</td>
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<tr>
<td>547.782</td>
<td>1,352.748</td>
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<tr>
<td>544.850</td>
<td>1,391.891</td>
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<tr>
<td>561.290</td>
<td>1,434.949</td>
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<tr>
<td>586.582</td>
<td>1,471.055</td>
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<td>609.176</td>
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<td>612.212</td>
<td>1,519.553</td>
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